Indian Statistical Institute, Bangalore
M.Math II Year, Second Semester
Semestral Examination, Back Paper Examination
Advanced Functional Analysis

Time: 3 hours | Instructor: T.S.S.R.K.Rao
Total Score : $10 \times 10 = 100$

Answer all questions. Show all work.

1. Let X, Y be topological vector spaces. Let $T : X \to Y$ be a linear map. Show that T is continuous if and only if it is bounded.

2. Let X be a $L\ C\ T\ V\ S$. Let $K \subset X$ be a compact set. Show that K is totally bounded.

3. Let X and Y be completely metrizable topological vector spaces. Let $T_n : X \to Y$ be a sequence of continuous linear maps such that $\lim\limits_{n\to\infty} T_n(x)$ exists for all $x \in X$. Show that $T(x) = \lim\limits_{n} T_n(x)$ is a bounded linear map.

4. State and prove the Banach-Alaoglu theorem for topological vector spaces.

5. Let K be a compact convex set in a $L\ C\ T\ V\ S,\ X$. Let $F \subset K$ be an extreme, convex, closed set. Show that F has an extreme point of K.

6. Let X be a Banach space and $(\Omega, \mathfrak{A}, \mu)$ a probability space. Let $f : \Omega \to X$ be a strongly $\mu-$ measurable function. Show that f in $a \cdot e$ separable valued.

7. Let X and f be an in question 6. Suppose $\int\limits_{\Omega} \| f(\omega) \| \, d\mu(\omega) < \infty$. Show that f is Bochner integrable.

8. Let $f : [0, 1] \to C[0, 1]$ be a measurable function such that inverse image of a Borel set is Borel. Is f strongly measurable? Justify your answer.

9. Let X be a Banach space and $f : [0, 1] \to X$ be a Bochner integrable function w.r.t. the Lebesgue measure. Show that $\lim\limits_{\lambda(E)\to 0} \int\limits_{E} f d\lambda = 0$.

10. State and prove the Pettis measurability theorem.